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Random magnetic fields and instantons in replica space

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Abstract. We study a ferromagnet in a random magnetic field using the replica method. We find that the mean field equations have localized solutions (i.e. instantons), which are not invariant under rotations in replica space. The relevance and the properties of these solutions are studied.

1. Introduction

The theoretical evaluation of the critical exponents of a ferromagnet with a random magnetic field is a difficult problem and new results have been obtained very slowly. There are a few safe statements. Naively we would expect the lower critical dimension to be 2 (Imry and Ma 1975). Indeed, if we try to reverse a large region there are two competing effects: the gain in energy due to the alignment with the random magnetic field, which scales as the volume to the power $\frac{1}{2}$; and the loss of energy due to the creation of an interface, which scales as the surface. For dimensions 2 or lower the two effects are comparable and no spontaneous magnetization should be present. On the the other hand, as soon as the dimension D is greater than 2, this effect should not spoil long-range order and a ferromagnetic transition (with peculiar critical exponents) should be present. This naive (but physically correct) argument was confirmed by a rigorous proof by Imbrie (1984).

On the other hand, a perturbative study of the transition has been done using the standard perturbative techniques of analysing fluctuations around the usual solution of the mean field equations. In this case it was shown that, as far as the leading infrared divergences were concerned, the strange phenomenon of a dimensional reduction was present and the critical exponents of the system in dimensions D were the same as those of the ferromagnetic system without random magnetic in dimension $d = D - 2$ (Young 1977).

This result would imply that the lower critical dimension is 3, at variance with the rigorous results. This paradox was solved in the following way. For a given magnetic field $h(x)$, the procedure of summing the leading infrared divergences is formally equivalent to determination free energies and expectation values. If the Hamiltonian in presence of the magnetic field has only one minimum, this procedure is described by the stochastic equation $\delta H/\delta\phi(x) = h(x)$ which has only one solution. Arriving at this point the dimensional reduction can be rigorously shown to be exact, by using of supersymmetrc arguments (Parisi and Sourlas 1979, Parisi 1982, Parisi 1987). More precise results could be obtained if we were to substitute the Hamiltonian with the free energy.

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However, as soon as the temperature is smaller than the transition temperature of the system without a magnetic field (the pure system in short), there are values of the magnetic field for which the free energy has more than one minimum and the stationary equations have more than one solution. In this situation there is no reason to believe that the supersymmetric approach should give the correct results and therefore the dimensional reduction is not applicable. The transition temperature of the system in a magnetic field is smaller than that in absence of a magnetic field, so that in the whole region near to the transition dimensional reduction does not hold. This is not surprising, because dimensional reduction completely misses the appearance of the Griffith's transition (Griffith 1969) which is present in the random system at the same temperature as the pure system. The existence of more than one solution is a non-perturbative phenomenon (with respect to the coupling constant g) and therefore we could expect that the deviations from the results of the dimensional reduction would be not be seen in the expansion of the exponents in powers of ϵ where $D = 6 - \epsilon$. In other words it is natural to conjecture that the difference between the true critical exponents and those of the dimensional reduction should be exponentially small when D goes to 6 (and perhaps proportionally to $\exp[-A/\epsilon]$). The precise procedure by which to compute such a difference is still a mystery.

It was shown that the existence of more than one solution to the stationary equations in the presence of the external field is related, in the replica approach, to the existence of new solutions to the mean field equations in replica space which are not invariant under translations and rotations in replica space (translation invariance and replica symmetry is recovered by considering the set of all possible solutions of this kind). It is possible that instantons in replica space may play a crucial role in understanding the deviations from the dimensional reduction for the critical exponents. The aim of this paper is to start to study in detail the properties of these instantons, which are quite peculiar. We do not attempt to investigate the behaviour of their contribution near the critical point, but we limit ourselves to investigating their properties in the low-temperature regime where perturbation theory may be used more safely. We hope that these results may be a first step towards a better understanding of the role of replica symmetry in the properties of random magnetic field ferromagnets.

The paper is organized as follows: in section 2 we introduce the model, the replica formalism and write down the equation for the instanton. In section 3 we study the solution of the equations in the previous section and we evaluate the instanton contribution to the free energy. In section 4 we evaluate the instanton contribution to the correlation functions.

2. The model

We consider the model which is described by the Hamiltonian

$$H = \int \mathrm{d}^d r[\tfrac{1}{2}(\nabla\phi(r))^2 + V(\phi(r)) + h(r)\phi(r)] \tag{1}$$

where ϕ is a continuous scalar field,

$$V(\phi) = \tfrac{1}{4}(\phi^2 - 1)^2 \tag{2}$$

and the random fields $h(r)$ are described by the Gaussian distribution

$$P(h) = \exp(-h^2/2h_0^2) \qquad h_0 \ll 1. \tag{3}$$

The free energy is calculating in terms of the replica approach:

$$-\beta F = \lim_{n\to 0} \frac{\langle\langle Z^n \rangle\rangle - 1}{n} \tag{4}$$

where $\langle\langle \ldots \rangle\rangle$ denotes the averaging over the random fields. For the replica partition function we obtain

$$\langle\langle Z^n \rangle\rangle = \int D\phi^a \exp\left\{ -\beta \int \mathrm{d}^d r \left[\tfrac{1}{2} \sum_{a=1}^{n} (\nabla\phi^a)^2 + \sum_{a=1}^{n} V(\phi^a) - \tfrac{1}{2}\beta h_0^2 \left(\sum_{a=1}^{n} \phi^a \right)^2 \right] \right\}. \tag{5}$$

We will be interested in the solutions with broken replica symmetry. Assuming that such solutions are on a finite distance from the replica symmetric one, the pattition function, equation (5), may be represented as follows

$$\langle\langle Z^n \rangle\rangle = \langle\langle Z^n \rangle\rangle_{\mathrm{RS}} + \langle\langle Z^n \rangle\rangle_{\mathrm{RSB}}. \tag{6}$$

The first term in equation (6) contains the summation over all the degrees of freedom near the replica symmetric states. Here we will study the second term, where the replica symmetry is broken.

Consider the situation when in the replica vector

$$\phi^a = (\phi^1, \phi^2, \ldots, \phi^n) \tag{7}$$

$(n-1)$ components are equal and only one component is different from them. In this case we are left with the two independent fields $\phi_1(r)$ and $\phi_2(r)$ and, in the limit $n \to 0$, we have $\sum_a \phi^a = \phi_1 + (n-1)\phi_2 \to \phi_1 - \phi_2$. For the partition function we then obtain

$$\langle\langle Z^n \rangle\rangle_{\mathrm{RSB}} \to n \int\int D\phi_1 D\phi_2 \exp\left\{ -\beta \int \mathrm{d}^d r [\tfrac{1}{2}(\nabla\phi_1)^2 - \tfrac{1}{2}(\nabla\phi_2)^2 + V(\phi_1) - V(\phi_2) - \tfrac{1}{2}\beta h_0^2 (\phi_1 - \phi_2)^2] \right\}. \tag{8}$$

The factor n appears due to the number of permutations. The saddle-point equations for the fields ϕ_1 and ϕ_2 are

$$\begin{aligned} -\Delta\phi_1 + V'(\phi_1) &= \gamma(\phi_1 - \phi_2) \\ -\Delta\phi_2 + V'(\phi_2) &= \gamma(\phi_1 - \phi_2) \end{aligned} \tag{9}$$

where $\gamma = \beta h_0^2$.

We are interested in the non-trivial smooth solutions with $\phi_1 \neq \phi_2$ and such that, at $r \to \infty$, they would coincide with the replica symmetric one: $\phi_1(\infty) = \phi_2(\infty)$. This way we will get a finite value for the integral in the exponent of equation (8), and correspondingly a finite contribution for the partition function.

Here we consider the zero temperature limit, when $\gamma \to \infty$. Redefining the fields:

$$\begin{aligned} \phi_1(r) &= \phi(r) + \frac{1}{\gamma}\psi(r) \\ \phi_2(r) &= \phi(r) \end{aligned} \tag{10}$$

in the lowest order in γ^{-1} we obtain the equations

$$\begin{aligned} -\Delta\phi(r) + V'(\phi(r)) &= \psi(r) \\ -\Delta\psi(r) + V''(\phi(r))\psi &= 0. \end{aligned} \tag{11}$$

These equations have simple non-trivial solution of the form

$$\psi(r) = -\frac{2}{r}\phi'(r) \tag{12}$$

where $\phi(r)$ is the solution of the equation

$$-\phi''(r) - \frac{d-3}{r}\phi'(r) + V'(\phi(r)) = 0. \tag{13}$$

This is the equation for the system without random fields with the shifted dimentionality $d \to d-2$. However, it does not mean (as will be shown later) that all the results are coming just from shifting dimentionality. In fact, in the solutions of the equations (9) there is a special kind of degeneracy. Taking

$$\begin{aligned} \phi_1 &= \phi + \frac{\lambda_1}{\gamma}\psi \\ \phi_2 &= \phi + \frac{\lambda_2}{\gamma}\psi \end{aligned} \tag{14}$$

where λ_1 and λ_2 are free parameters, we obtain the equations

$$\begin{aligned} -\Delta\phi + V'(\phi) &= (\lambda_1 - \lambda_2)\psi \\ -\Delta\psi + V''(\phi)\psi &= 0. \end{aligned} \tag{15}$$

Taking $\lambda_1 - \lambda_2 = 1$ we again obtain the equations (11). Therefore, the solution of equations (9) in the lowest order in γ^{-1} are

$$\begin{aligned} \phi_1(r) &= \phi(r) + \frac{\lambda}{\gamma}\psi(r) \\ \phi_2(r) &= \phi(r) + \frac{\lambda-1}{\gamma}\psi(r) \end{aligned} \tag{16}$$

where the functions $\phi(r)$ and $\psi(r)$ are given by equations (12) and (13), and λ is a free parameter.

It will be shown in the next section that the solution of equation (13) for the function $\phi(r)$ also contains an additional parameter which is $\phi(r=0) \equiv \phi_0$. This is the instanton-like solution, and it has a finite spatial size $L(\phi_0)$. Therefore, to calculate the partition function, equation (8), one should take the instanton solution for the fields ϕ_1 and ϕ_2, equation (16), and integrate over the remaining degrees of freedom, which are λ, ϕ_0 and the position of the centre of the soliton.

After some algebra we obtain

$$\langle\langle Z^n \rangle\rangle_{\mathrm{RSB}} \simeq n \int \mathrm{d}\phi_0 \frac{V}{L(\phi_0)^d} \exp\left\{-\frac{E_0(\phi_0)}{2h_0^2}\right\} \times \int \mathrm{d}\lambda \exp\left\{-\frac{E_2(\phi_0)}{2h_0^2\gamma^2}\lambda^2 - \frac{E_3(\phi_0)}{6h_0^2\gamma^3}\lambda^3\right\} \tag{17}$$

where V is a volume of the system and

$$E_0 = \int \mathrm{d}^d r \psi(r)^2 = 4 \int \frac{\mathrm{d}^d r}{r^2} (\phi'(r))^2 \tag{18}$$

$$E_2 = \int \mathrm{d}^d r V'''(\phi(r))\psi(r)^3 = -48 \int \frac{\mathrm{d}^d r}{r^3} \phi(r)(\phi'(r))^3 \tag{19}$$

$$E_3 = \int \mathrm{d}^d r V''''(\phi(r))\psi(r)^4 = 96 \int \frac{\mathrm{d}^d r}{r^4} (\phi'(r))^4. \tag{20}$$

The integration over the parameter λ goes near the minimum of the function in the exponent of equation (17). If E_0, E_2 and E_3 are converging and if $E_2 > 0$, which will be shown to be the case for $d < 3$, the minimum is at $\lambda = 0$, and the integration over λ gives

$$\langle\langle Z^n \rangle\rangle_{\mathrm{RSB}} \simeq nV\beta h_0^3 \int \mathrm{d}\phi_0 \left[\exp\left\{-\frac{E_0(\phi_0)}{2h_0^2}\right\} \Big/ L(\phi_0)^d \sqrt{E_2(\phi_0)}\right]. \tag{21}$$

Correspondingly, for the density of the free energy we obtain the following finite contribution:

$$f_{\mathrm{RSB}} \simeq h_0^3 \int \mathrm{d}\phi_0 \left[\exp\left\{-\frac{E_0(\phi_0)}{2h_0^2}\right\} \Big/ L(\phi_0)^d \sqrt{E_2(\phi_0)}\right]. \tag{22}$$

It can be shown (see appendix) that the contribution of the higher orders of the replica symmetry breaking in the replica vector $\phi^a(r)$, equation (7), does not exist. Therefore, result (22) is the only contribution to the zero-temperature free energy from the states with broken replica symmetry.

3. The solutions

(i) For $d > 3$ the smooth solutions of the equation

$$\phi'' + \frac{d-3}{r}\phi' = \phi(\phi^2 - 1) \tag{23}$$

with $\phi(\infty) = 1$ and $\phi(0) \neq 1$ do not exist.

(ii) For $d = 3$ the solution is

$$\phi(r) = \tanh((r - L)/\sqrt{2}) \tag{24}$$

where

$$-\tanh(L/\sqrt{2}) = \phi(0) \equiv \phi_0. \tag{25}$$

However, it can be easily shown that the factors E_2 and E_3, equations (19) and (20), are divergent at $r \to 0$.

(iii) Consider the case $2 < d < 3$:

$$d = 3 - \epsilon \qquad 0 < \epsilon < 1. \tag{26}$$

The asymptotics of the solution of equation (13) are

$$\phi(r) \simeq \begin{cases} \frac{\epsilon}{1+\epsilon} a(r/\epsilon)^{1+\epsilon} + \phi_0 & \text{for } r \ll \epsilon \\ \tanh((r - L)/\sqrt{2}) & \text{for } r \gg \epsilon. \end{cases} \tag{27}$$

Note that the case $\epsilon = 1 (d = 2)$ needs special consideration since the asymptotics, equation (27), are not valid in this case, and therefore it is not considered here.

The conditions for a smooth connection of these two asymptotics at $r \sim \epsilon$ give:

$$\begin{aligned} &\frac{\epsilon}{1+\epsilon} a + \phi_0 \simeq \tanh\left(\frac{\epsilon - L}{\sqrt{2}}\right) \\ &a \simeq \frac{1}{\sqrt{2}}\left(1 - \tanh^2\left(\frac{\epsilon - L}{\sqrt{2}}\right)\right). \end{aligned} \tag{28}$$

These two equations define the parameters a and L as functions of ϕ_0.

It will become clear from what follows that the main contribution to the free energy, equation (22), comes from the vicinity of the point $\phi_0 = -1$. If $(1 - \phi_0^2) \ll 1$, we obtain

$$\begin{aligned} &a(\phi_0) \simeq a_0(1 - \phi_0^2) \\ &L(\phi_0) \simeq \frac{1}{\sqrt{2}} \log \frac{1}{1 - \phi_0^2} \end{aligned} \tag{29}$$

where $a_0 = (1+\epsilon)[\sqrt{2}(1+\epsilon) + 2\epsilon]^{-1}$ is a numerical factor.

Now one can easily estimate E_0, E_2 and E_3, equations (18)–(20),

$$E_0(\phi_0) \simeq a_1 \epsilon^{1-\epsilon}(1 - \phi_0^2)^2 + a_2\left(\log \frac{1}{1 - \phi_0^2}\right)^{-\epsilon} \tag{30}$$

$$E_2(\phi_0) \simeq a_3 \epsilon^{-1-\epsilon}(1 - \phi_0^2)^3 + a_4\left(\log \frac{1}{1 - \phi_0^2}\right)^{-2-\epsilon}. \tag{31}$$

The factor E_3 converges only for $\epsilon > \frac{1}{3}$:

$$E_3(\phi_0) \simeq a_5 \frac{\epsilon^{-1-\epsilon}}{3\epsilon - 1}(1 - \phi_0^2)^4 + a_6\left(\log \frac{1}{1 - \phi_0^2}\right)^{-2-\epsilon}. \tag{32}$$

Here $a_1, a_2, \ldots, a_6$ are irrelevant numerical factors.

Using equations (30)–(32) and integrating over ϕ_0 for the free energy, equation (22), we finally obtain the result

$$f_{\mathrm{RSB}} \sim h_0^4\left(\log \frac{1}{h_0}\right)^{-2-\frac{3}{2}\epsilon} \exp\left\{-\frac{a_2}{2h_0^2}\left(\log \frac{1}{h_0}\right)^{-\epsilon}\right\} \tag{33}$$

where it is assumed that $\epsilon > \frac{1}{3}$.

4. Correlation function

In terms of the replica approach the correlation function $G(r) = \overline{\langle\phi(0)\phi(r)\rangle}$ can be represented as follows

$$G(r) = \lim_{n\to 0}\int D\phi^a \phi^1(0)\phi^1(r)\exp\{-\beta H[\phi^a]\} \tag{34}$$

or

$$G(r) = G(r)_{\mathrm{RS}} + \int D\phi_1 D\phi_2(\phi_1(0)\phi_1(r) - \phi_2(0)\phi_2(r))\exp(-\beta H[\phi_1,\phi_2]) \tag{35}$$

where the first term is the replica symmetric contribution and $H[\phi_1,\phi_2]$ is the Hamiltonian in equation (8).

Following the calculations from section 2, for the second term in the equation (35), which is the contribution coming from the replica symmetry breaking, we obtain

$$G(r)_{\mathrm{RSB}} = h_0\int \mathrm{d}\phi_0 \frac{\exp(-(E_0/2h_0^2))}{\sqrt{E_2}}\int \mathrm{d}^d R(\phi(\boldsymbol{R})\psi(\boldsymbol{R}+\boldsymbol{r}) + \psi(\boldsymbol{R})\phi(\boldsymbol{R}+\boldsymbol{r})). \tag{36}$$

The second integration in equation (36) is the integration over the location of the soliton.

Using the solutions for the functions $\phi(r)$ and $\psi(r)$, equations (27) and (12), and the estimations for E_0 and E_2, equations (30) and (31), for $1 \ll r \ll \log 1/h_0$ we obtain ($d = 3-\epsilon$, $1/3 < \epsilon < 1$):

$$\begin{aligned} G(r)_{\mathrm{RSB}} &= -c_1 h_0^2 r^{2-\frac{1}{2}\epsilon}\exp\left\{-\frac{a_1\epsilon^{1-\epsilon}}{2h_0^2}\exp(-r) - \frac{a_2}{2h_0^2 r^\epsilon}\right\} \\ &\quad - c_2 h_0^2\left(\log\frac{1}{h_0}\right)^{1-\frac{1}{2}\epsilon}\exp\left\{-\frac{a_2}{2h_0^2}(\log\frac{1}{h_0})^{-\epsilon}\right\} \end{aligned} \tag{37}$$

where c_1 and c_2 are numerical factors.

For $r \gg \log 1/h_0$ the RSB part of the correlation function becomes independent of r:

$$G(r\to\infty)_{\mathrm{RSB}} = -c_1 h_0^2\left(\log\frac{1}{h_0}\right)^{2-\frac{1}{2}\epsilon}\exp\left\{-\frac{a_2}{2h_0^2}\left(\log\frac{1}{h_0}\right)^{-\epsilon}\right\}. \tag{38}$$

5. Conclusions

In the present paper we have demonstrated that in the random field systems there exist localized instanton-like states giving finite contributions to the free energy and to the correlation functions. These states are not invariant under rotations in the replica space, they have finite energy and finite size, and they are clearly beyond any kind of a perturbation theory.

However, several questions remain to be answered. It is not quite clear what happens with the obtained replica instantons at finite temperatures and in particular near the transition point. The two-dimensional system appeared to be beyond the present study and it requires special consideration. A more general problem is to understand what is the interrelation (if any) between the Griffith's singularities and the obtained instanton states.

Appendix

Higher orders of RSB in the present approach mean that in the replica vector $\phi^a (a = 1, 2, ..., n)$ $m > 1$ components are different and the rest $(n - m)$ components are equal:

$$\phi^a = \begin{cases} \phi_a(r) & \text{if } a = 1,2,\ldots,m \\ \phi_0(r) & \text{if } a = m+1,\ldots,n. \end{cases} \tag{39}$$

For the replica partition function we obtain

$$\langle\langle Z^n \rangle\rangle^{(m)}_{\mathrm{RSB}} = C^n_m \int \left(\prod_{a=1}^m D\phi_a\right) D\phi_0 \exp\left\{ -\beta \int \mathrm{d}^d r \left[\frac{1}{2}\sum_{a=1}^m ((\nabla\phi_a)^2 - (\nabla\phi_0)^2) + \sum_{a=1}^m (V(\phi_a) - V(\phi_0)) - \frac{1}{2}\left(\sum_{a=1}^m (\phi_a - \phi_0)\right)^2 \right]\right\}. \tag{40}$$

Here C^n_m is the number of permutations. In the limit $n \to 0$,

$$C^n_m \to n\frac{(-1)^m}{m}. \tag{41}$$

The corresponding saddle-point equations are

$$\begin{aligned} -\Delta\phi_a + V'(\phi_a) &= \gamma \sum_{a=1}^m (\phi_a - \phi_0) \\ -\Delta\phi_0 + V'(\phi_0) &= \gamma \sum_{a=1}^m (\phi_a - \phi_0). \end{aligned} \tag{42}$$

In the lowest order in γ^{-1} the solutions of these equations are

$$\begin{aligned} \phi_a(r) &= \phi(r) + \frac{\lambda_a}{\gamma}\psi(r) \\ \phi_0(r) &= \phi(r) + \frac{\lambda}{\gamma}\psi(r) \end{aligned} \tag{43}$$

where $\phi(r)$ and $\psi(r)$ are the solutions (12) and (13) of equations (11), and the parameters λ_a and λ are bounded by the condition

$$\sum_{a=1}^m (\lambda_a - \lambda) = 1. \tag{44}$$

The corresponding contribution to the partition function is

$$n\frac{(-1)}{m} \int \mathrm{d}\phi_0 \frac{V}{L(\phi_0)^d} \exp\left\{-\frac{E_0(\phi_0)}{2h_0^2}\right\} \int \left(\prod_{a=1}^m \mathrm{d}\lambda_a\right) \mathrm{d}\lambda \times \exp\left\{-\frac{E_2}{6\gamma^2 h_0^2}\sum_{a=1}^m (\lambda_a^3 - \lambda^3) - \frac{E_3}{24\gamma^3 h_0^2}\sum_{a=1}^m (\lambda_a^4 - \lambda^4)\right\} \times \delta\left(\sum_{a=1}^m (\lambda_a - \lambda) - 1\right). \tag{45}$$

Introducing

$$t_a = m\lambda_a - \sum_{b=1}^{m} \lambda_b \qquad u = \sum_{b=1}^{m} \lambda_b \tag{46}$$

where $\sum_a^m t_a \equiv 0$, instead of the integration over λ's in equation (45) we obtain

$$\int \mathrm{d}u \left(\prod_{a=1}^{m} \mathrm{d}t_a\right) \exp\left\{-\frac{E_3}{6\gamma^2 h_0^2}u^3 + \frac{1}{2\gamma^2 m^2}\left(\frac{E_2}{h_0^2} + \frac{E_3}{2\gamma h_0^2 m^2}\sum_{a=1}^{m}(t_a^2 - 1)\right)u^2 \right.$$
$$- \frac{1}{2\gamma^2 m^3 h_0^2}\left[E_2 \sum_{a=1}^{m}(t_a^2 - 1) + \frac{E_3}{3\gamma m}\sum_{a=1}^{m}(t_a^3 + 1)\right]u$$
$$\left. - \frac{E_3}{24\gamma^3 h_0^2 m^4}\sum_{a=1}^{m} t_a^4 - \frac{E_2}{6\gamma^2 m^3 h_0^2}\sum_{a=1}^{m} t_a^3\right\} \delta\left(\sum_{a=1}^{m} t_a\right). \tag{47}$$

Leaving only the leading orders in γ^{-1} and integrating over t_a' we obtain

$$\frac{1}{\sqrt{m}} \int \mathrm{d}u \exp\left\{-\frac{E_3}{6\gamma^3 h_0^2}u^3 - \frac{E_2}{2\gamma^2 m^2 h_0^2}u^2 \right.$$
$$\left. - \frac{m-1}{2}\log\left[\frac{E_2}{\gamma m^3 h_0^2}u + \frac{E_3}{2\gamma^2 m^4 h_0^2}u^2\right]\right\}. \tag{48}$$

This integral is pathological for any $m \neq 1$.

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